Error correction in the early-FTQC regime

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Partially Fault-Tolerant Quantum Computing Architecture with Error-Corrected Clifford Gates and Space-Time Efficient Analog Rotations

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Quantum computers are expected to drastically accelerate certain computing tasks versus classical computers. Noisy intermediate-scale quantum (NISQ) devices, which have tens to hundreds of noisy physical qubits, are gradually becoming available, but it is still challenging to achieve useful quantum advantages in meaningful tasks. On the other hand, full fault-tolerant quantum computing (FTQC) based on quantum error correction code remains far beyond realization due to its extremely large requirement of high-precision physical qubits. In this study, we propose a quantum computing architecture to close the gap between NISQ and FTQC architectures. Our architecture is based on erroneous arbitrary rotation gates and error-corrected Clifford gates implemented by lattice surgery. We omit the typical distillation protocol to achieve direct analog rotations and small qubit requirements, and minimize the remnant errors of the rotations by a carefully designed state injection protocol. Our estimation based on numerical simulations shows that for early-FTQC devices that consist of 10^4 physical qubits with physical error probability $p = 10^{-4}$, we can perform roughly 1.72×10^7 Clifford operations and 3.75×10^4 arbitrary rotations on 64 logical qubits. Such computations cannot be realized by the existing NISQ and FTQC architectures on the same device, as well as classical computers. We hope that our proposal and the corresponding development of quantum algorithms based on it will bring new insights into the realization of practical quantum computers in the future.

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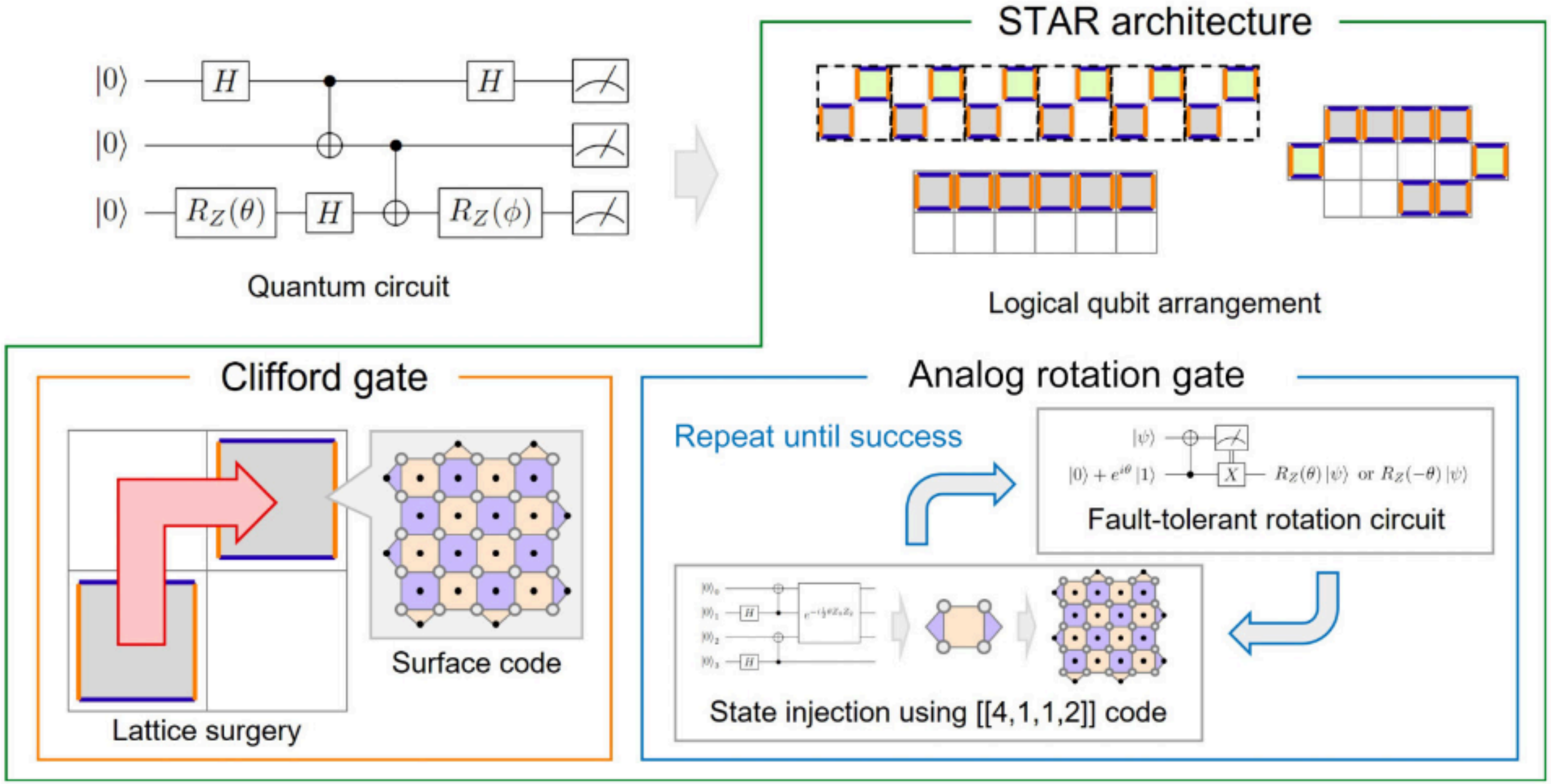
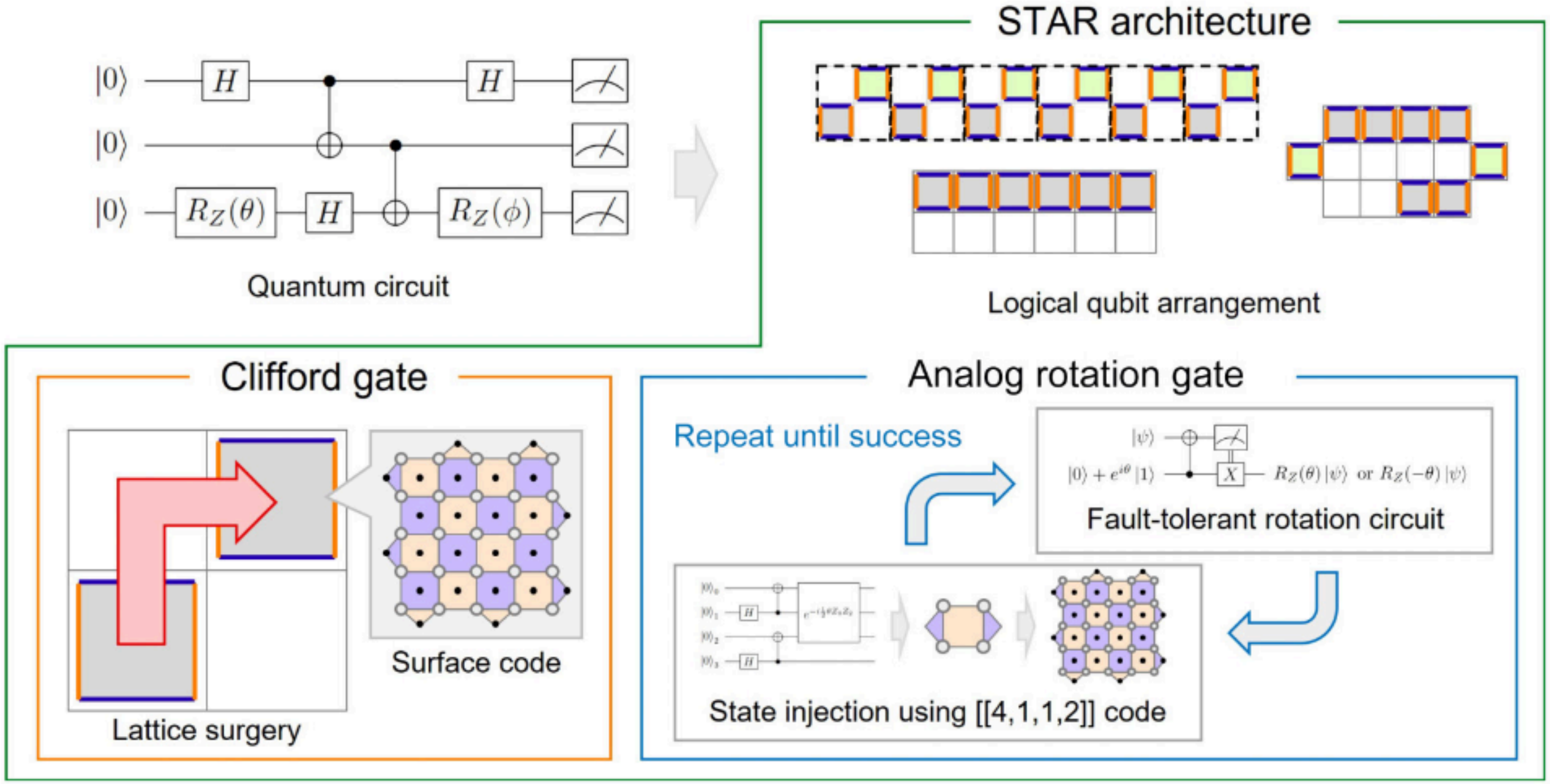
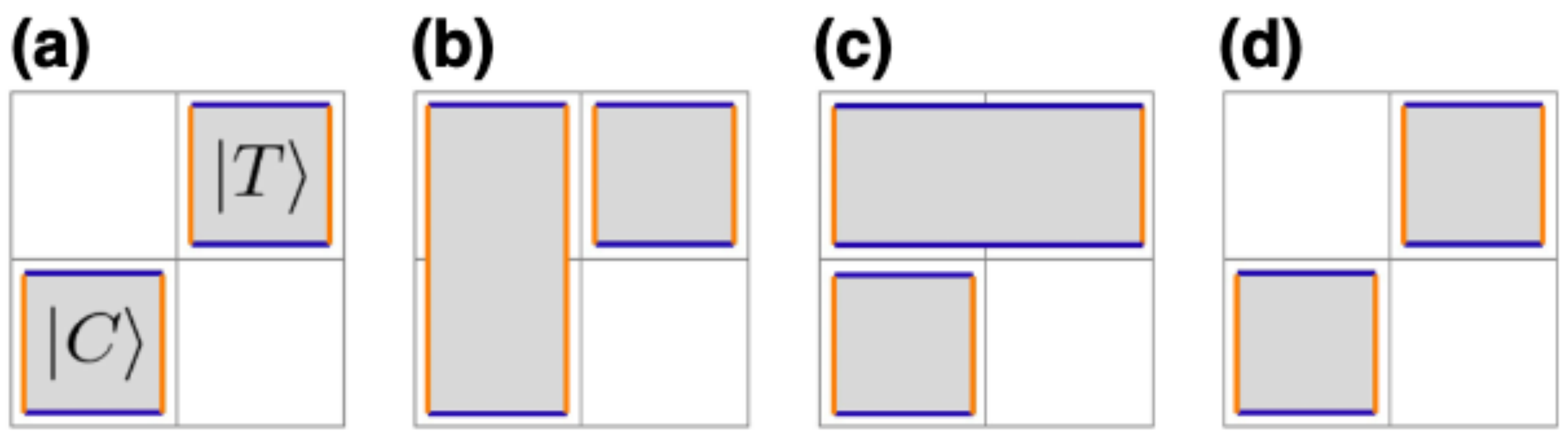
STAR architecture



STAR architecture



CNOT



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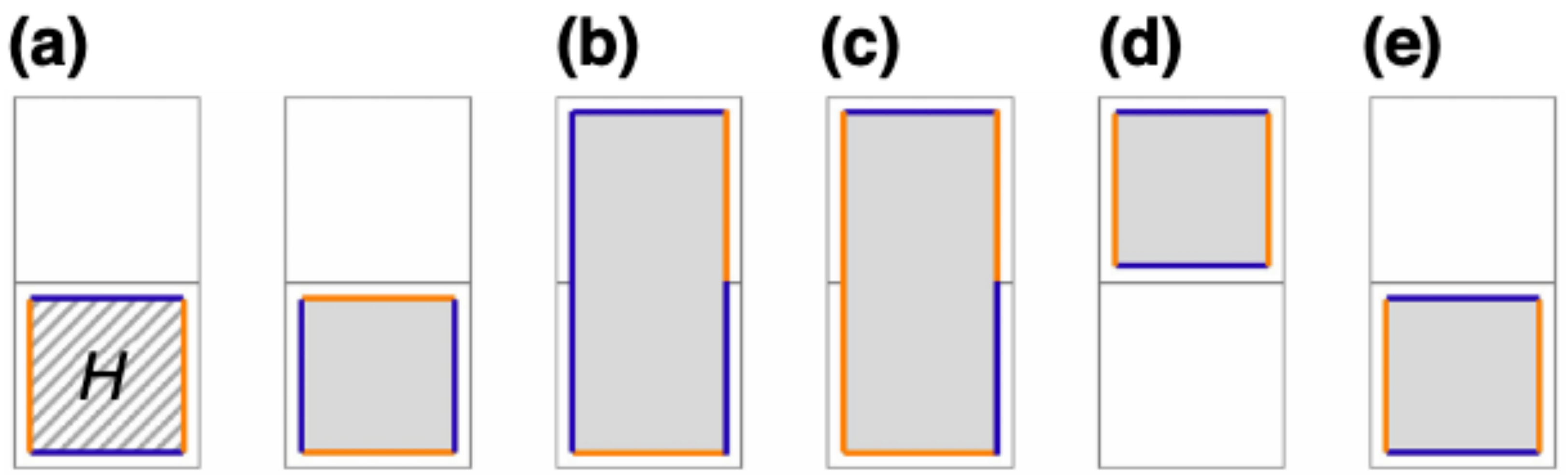
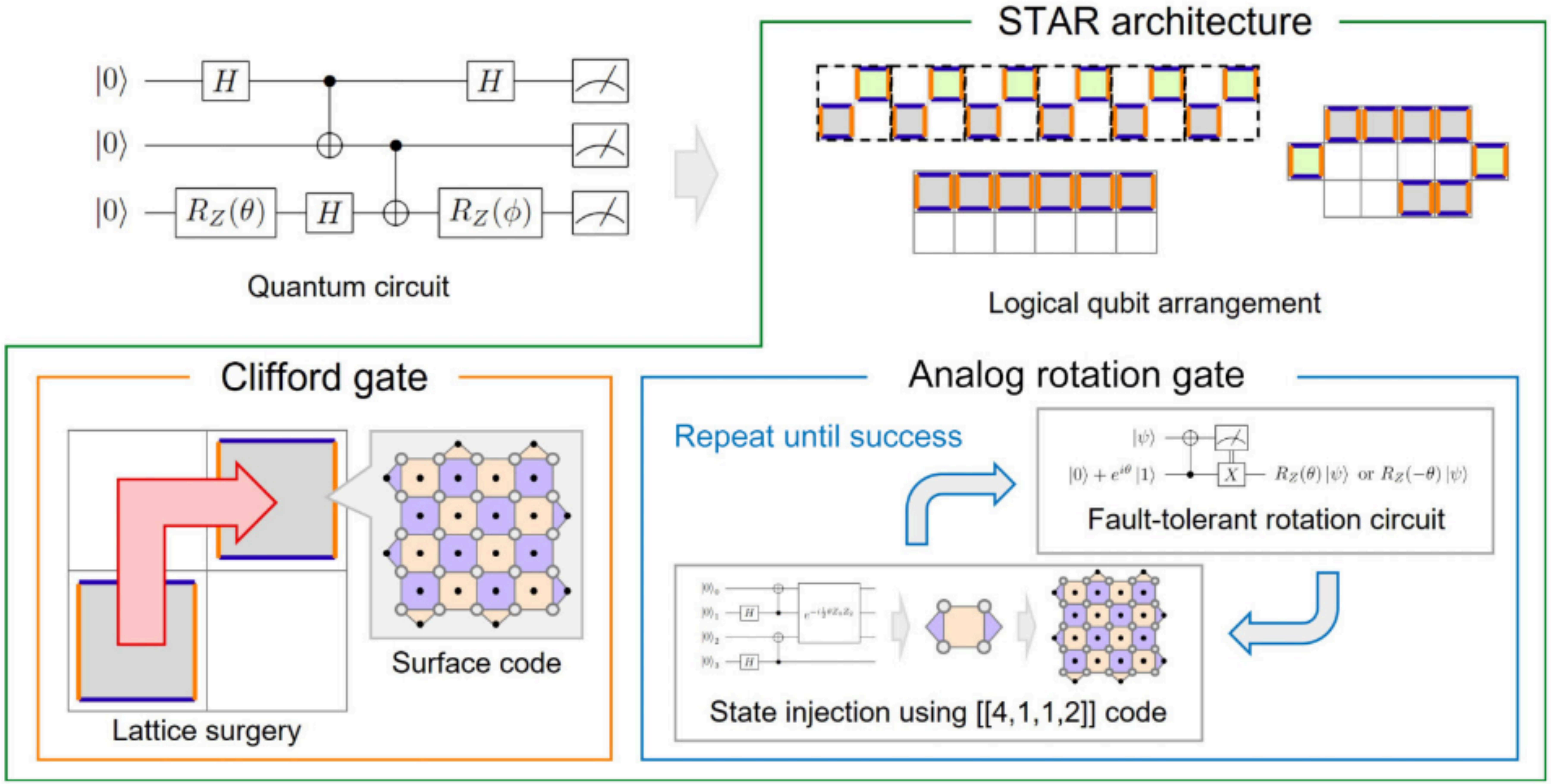
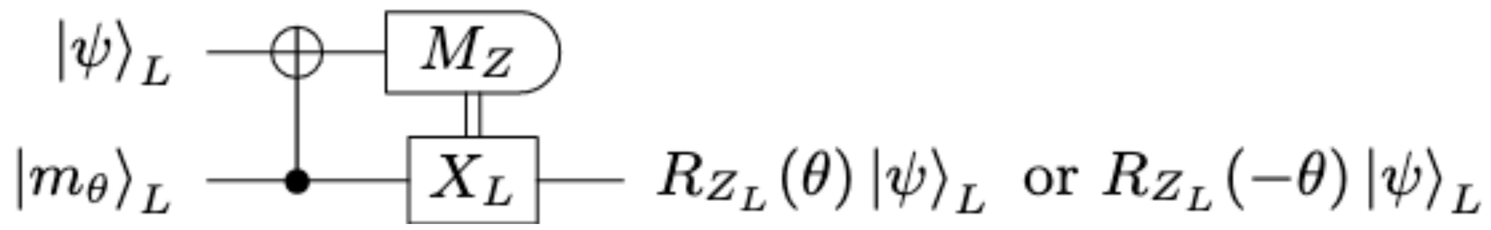


STAR architecture



Analog Z rotation gate

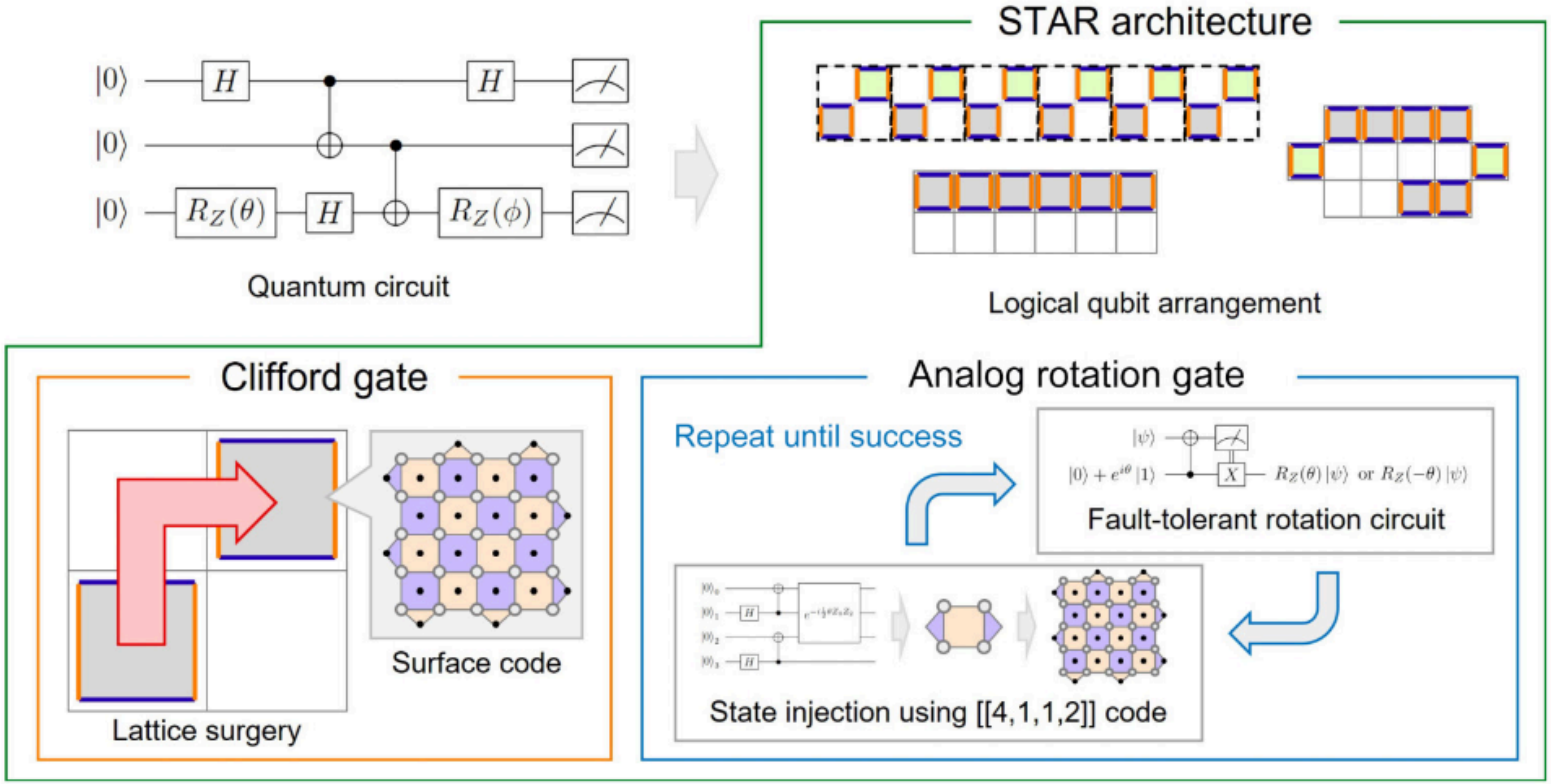
$$|m_\theta\rangle \equiv R_Z(\theta)\,|+\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\theta/2}\,|0\rangle + e^{+i\theta/2}\,|1\rangle\right)$$



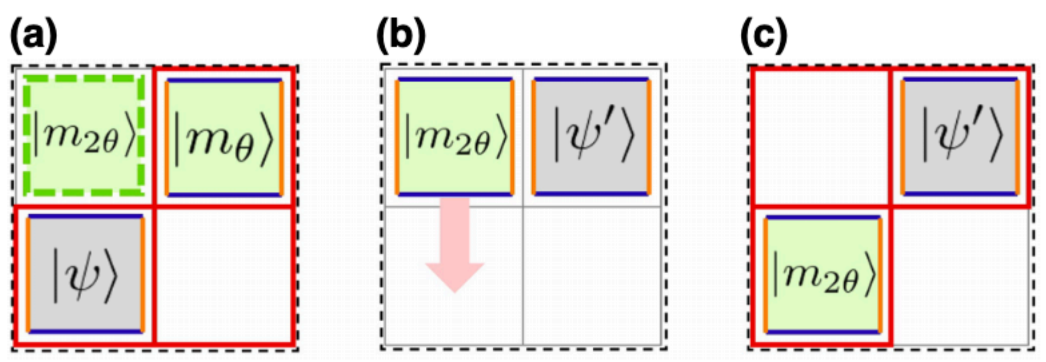
RUS

$$1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + \cdots = \sum_{i=1}^{\infty} \frac{n}{2^n} = 2. \quad (1)$$

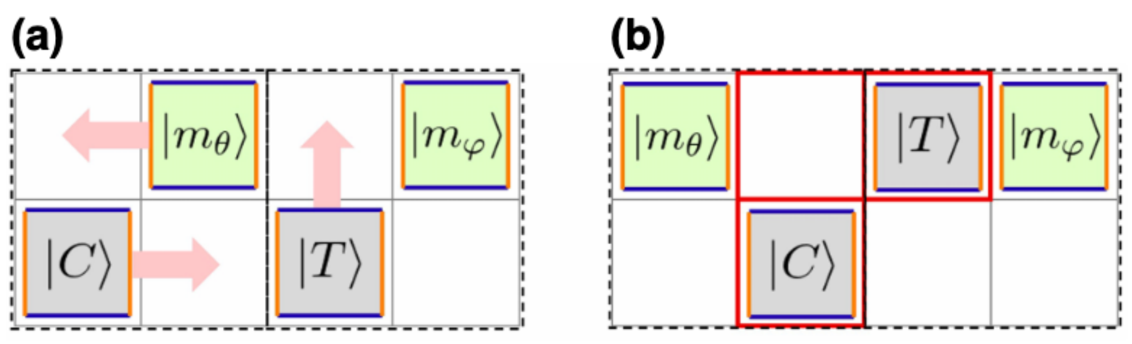
STAR architecture



RUS protocol within data unit



Direct logical CNOT operation



Applications

One promising application of the STAR architecture is a quantum many-body simulation because the time-evolution operator can be implemented easily by analog rotation gates. For example, we consider a two-dimensional Hubbard model with $N = N_x N_y$ sites. By using the Jordan-Wigner transformation and snake-shaped indexing [57], we can write the Hamiltonian in terms of Pauli operators as

$$H = -\frac{t}{2}\sum_{\langle i,j \rangle}(X_i X_j + Y_i Y_j) Z^{\leftrightarrow}_{i,j} + \frac{U}{4}\sum_{i=0}^{N-1} Z_i Z_{2N-1-i} - \frac{U}{4}\sum_{i=0}^{2N-1} Z_i, \tag{21}$$

$$Z^{\leftrightarrow}_{i,j} = \prod_{k=i+1}^{j-1} Z_k, \tag{22}$$

We estimate how many Trotter steps our architecture can perform in the two-dimensional Hubbard model simulation. Equation (22) consists of $8N + N + 2N = 11N$ terms; thus, its time evolution of a single Trotter step requires $11N$ arbitrary rotation gates. If we choose the $d = 7$ ($d = 9$) architecture and fully allocate data logical qubits for N sites, we can simulate $N = 64/2 = 32$ ($N = 37/2 = 18$) sites. The actual number of rotation gates per Trotter step is $11 \times 32 = 352$ ($11 \times 18 = 198$). Therefore, we can simulate real-time dynamics with $3.75 \times 10^4/352 \approx 107$ ($3.75 \times 10^4/198 \approx 189$) Trotter steps for this system.

Another promising application is the QAOA [61] for solving binary optimization problems. For example, we consider the maximum-cut problem for a graph with N nodes. The problem Hamiltonian is given as

$$H_C = -\frac{1}{2}\sum_{i \neq j}(1 - Z_i Z_j). \tag{23}$$

To obtain the ground state of H_C, we consider the QAOA ansatz state

$$|\gamma, \beta\rangle = e^{-i\beta_{p-1}H_B} e^{-i\gamma_{p-1}H_C} \cdots e^{-i\beta_0 H_B} e^{-i\gamma_0 H_C} \times H^{\otimes N} |0\rangle^{\otimes N}, \tag{24}$$

where

$$H_B = \sum_{j=0}^{N-1} X_j \tag{25}$$

and $\gamma = (\gamma_0, \ldots, \gamma_{p-1})$ and $\beta = (\beta_0, \ldots, \beta_{p-1})$ are optimization parameters. They are optimized to minimize an expectation value $\langle \gamma, \beta | H_C | \gamma, \beta \rangle$. The ansatz state contains $p(N + ((N(N-1))/2))$ arbitrary rotations in total. If we choose the $d = 7$ ($d = 9$) architecture and set $N = 64$

($N = 37$), we can take the depth of the ansatz as $p = 3.75 \times 10^4/2080 \approx 18$ ($3.75 \times 10^4/703 \approx 53$). Note that higher-order binary optimization problems can be directly solved in the STAR architecture without any reduction to quadratic unconstrained binary optimization problems, because the Clifford gates are almost error-free.