Paper overview

The unbearable hardness of deciding about magic

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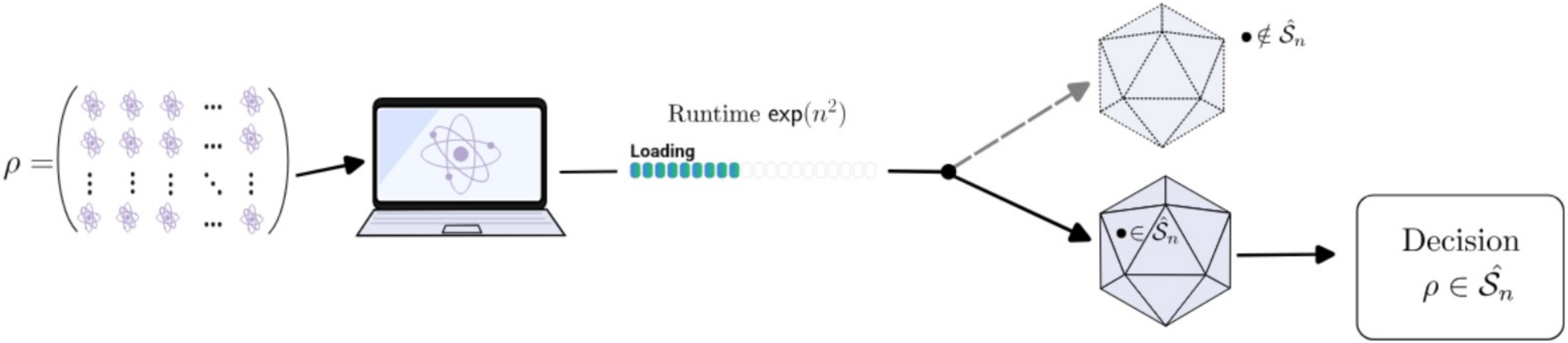
What is magic?

Two key ressources :

- Entanglement : (low vs high entanglement)
  Classical tool : Tensor networks
- Magic : (Clifford vs Universal)
  Classical tool : Stabilizer formalism
- Both? : (relatively low entanglement & « magic »)
  Stabilizer TN (http://arxiv.org/abs/2403.08724)

Theorem

Theorem 1 (Membership in the stabilizer polytope is hard. Informal of Theorem 6 and Lemma 11)**.** *Deciding whether a state ρ lies in the stabilizer polytope $\hat{\mathcal{S}}$, or is ε-far from every state in $\hat{\mathcal{S}}$, belongs to the complexity class* QP^2 *for any* $\varepsilon = 1/\mathsf{poly}(d)$.



Proof sketch

Proof sketch. The proof has two parts: showing that the decision problem lies in QP^2, and that it does not belong to $\mathsf{QP}^{2-\eta}$ for any $\eta > 0$. Membership in QP^2 follows from computing the known magic monotone, the *robustness of magic*, which can be evaluated in time $\mathsf{exp}(\log^2 d)$ using the argument in Ref. [21], rigorously refined in Lemma 11. The second, more technical part encodes an instance of the polytope membership problem as a 3-SAT instance in $O(\log^2 d)$ variables. By the ETH, this cannot be solved in time $\mathsf{exp}(\log^{2-\eta} d)$ for any $\eta > 0$. □

Implications

Quantifying magic

Corollary 1 (No efficient magic monotone). *The computation of any magic monotone $\mathcal{M}$ requires time* $\mathsf{exp}(\log^2 d)$.

Witnessing magic

Theorem 2 (Finding witnesses is hard. Informal of Theorem 7 and Lemma 12). *Given a Hermitian operator W, deciding whether $\mathrm{tr}(W\sigma) \leq 0$ for all $\sigma \in \hat{\mathcal{S}}$ (i.e., W is a valid witness) or there exists $\tau \in \hat{\mathcal{S}}$ such that $\mathrm{tr}(W\tau) > 0$ (i.e., W it is not a witness) belongs to the complexity class QP^2.*

Classical-quantum boundary

Theorem 3 (Tracing the classical-quantum boundary is hard. Informal of Theorem 8). *For any $\varepsilon = 1/\mathsf{poly}(d)$, the problem of deciding whether a state ρ lies in $\hat{\mathcal{S}}_t$ or is ε-far from every state in $\hat{\mathcal{S}}_t$ satisfies:*

- *It belongs to the complexity class QP^2 for any $t < \log n$.*
- *For any $t = O(\log n)$ lies at least in QP^2 and at most in $\mathsf{QP} = \bigcup_k \mathsf{QP}^k$.*

Implications

Classical simulation of noisy quantum circuits

Corollary 2 (Deciding classical simulation of noisy circuits is hard. Informal of Corollaries 4 and 5). *Given a quantum channel $\mathcal{E}$, deciding whether $\mathcal{E}$ is completely t-doped stabilizer preserving or not does not belong to the complexity class $\mathsf{QP}^{2-\eta}$ for any $\eta > 0$. In particular, this problem is much harder than brute-force simulating the process $\mathcal{E}$ on a classical device.*

Magic-state distillation

Corollary 3 (Super-polynomial time magic-state distillation. Informal of Corollary 6). *There exists a (possibly mixed) non-stabilizer state ρ on n qubits such that any stabilizer-based distillation protocol that can be found and executed within polynomial time in d produces at most a super-polynomially small $\mathsf{exp}(-\log^2 d)$ fraction of high-fidelity magic states.*