Quantum Automated Theorem Proving

Quantum automated theorem proving

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Automated theorem proving, or more broadly automated reasoning, aims at using computer programs to automatically prove or disprove mathematical theorems and logical statements. It takes on an essential role across a vast array of applications and the quest for enhanced theorem-proving capabilities remains a prominent pursuit in artificial intelligence. Here, we propose a generic framework for quantum automated theorem proving, where the intrinsic quantum superposition and entanglement features would lead to potential advantages. In particular, we introduce quantum representations of knowledge bases and propose corresponding reasoning algorithms for a variety of tasks. We show how automated reasoning can be achieved with quantum resolution in both propositional and first-order logic with quadratically reduced query complexity. In addition, we propose the quantum algebraic proving method for geometric theorems, extending Wu's algebraic approach beyond the classical setting. Through concrete examples, including geometry problems from the International Mathematical Olympiad, we demonstrate how a quantum computer may prove geometric theorems with quadratic better query complexity. Our results establish a primary approach towards building quantum automatic theorem provers, which would be crucial for practical applications of both near-term and future quantum technologies.

Natives of the Pincus planet*

The problem

Travellers to the planet Pincus have observed that all its **healthy** natives are **noisy**, unless they are **fearful**. Those who are **fearful** and **quiet** are **happy**, as are those who are **healthy** and **noisy**. The **happy** and **quiet** natives are **healthy**, but those who are **fearful** and **healthy** are **unhappy**. Finally, even though the **unhappy** and **unhealthy** natives are always **afraid**, the **fearful** and **noisy** ones are **healthy.**

Fearful / Courageous   Happy / Unhappy   Unhealthy / Healthy   Noisy / Quiet

What can we deduce about the natives of the Pincus planet?

*Problem adapted from *The Art of Computer Programming*, Volume 4, Donald E. Knuth

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Natives of the Pincus planet*

The **conjunction** (x **and** y) : $x \wedge y$ is true if x is true **and** y is true

The **disjunction** (x **or** y) : $x \vee y$ is true if x is true **or** y is true

The **negation** (**not** x) : $\neg x$ is true if x is false (also written $\bar{x}$)

$$f(x_0, x_1, x_2, x_3) =$$

$$(x_2 \vee x_0 \vee x_3) \wedge (\bar{x}_0 \vee x_3 \vee x_1) \wedge$$

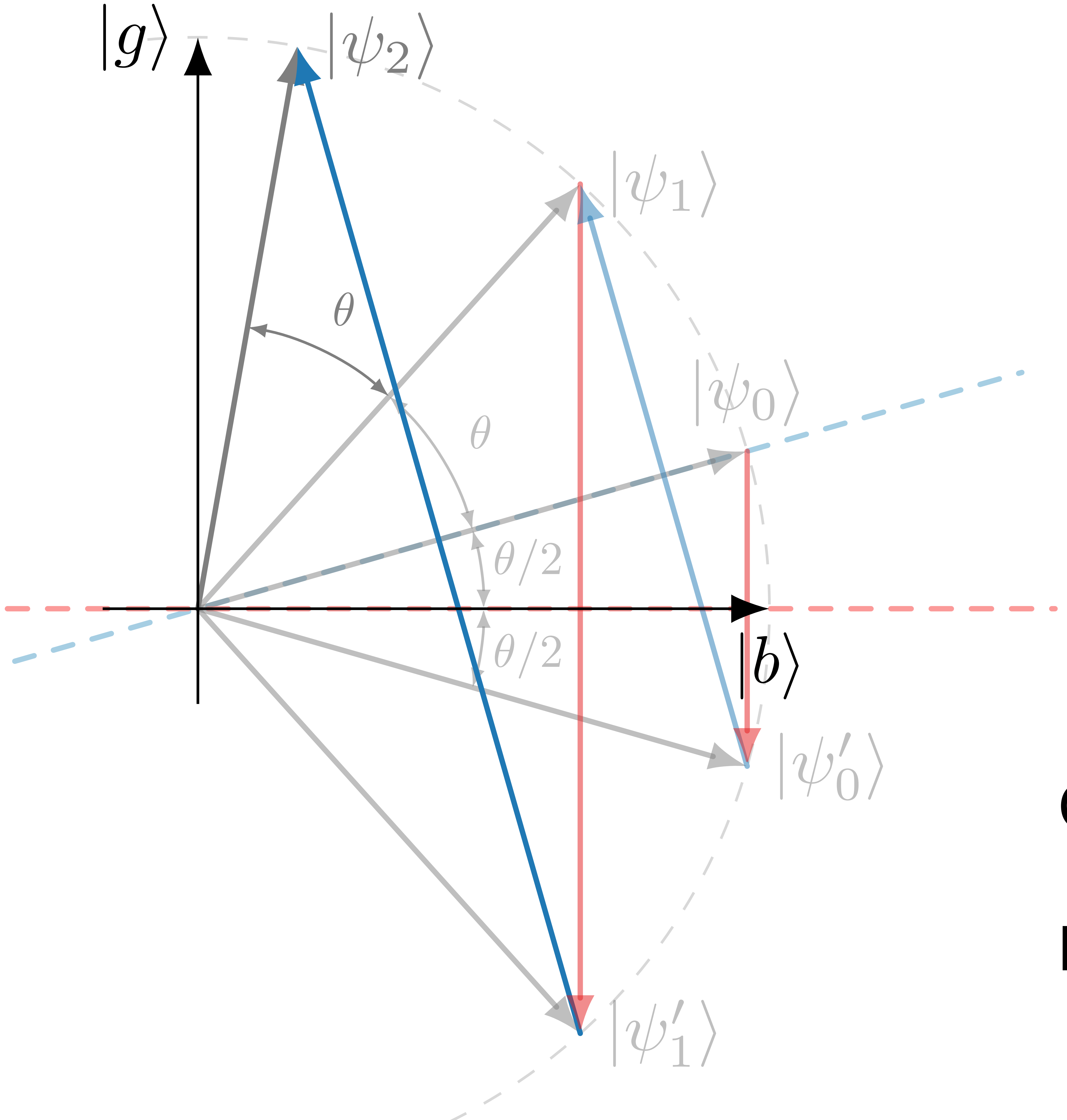
$$(x_2 \vee \bar{x}_3 \vee x_1) \wedge (\bar{x}_1 \vee x_3 \vee \bar{x}_2) \wedge$$

$$(\bar{x}_0 \vee x_2 \vee \bar{x}_1) \wedge (x_1 \vee \bar{x}_2 \vee x_0) \wedge$$

$$(\bar{x}_0 \vee \bar{x}_3 \vee \bar{x}_2$$

		TRUE	FALSE
🫣	x_0	Fearful	Courageous
😀	x_1	Happy	Unhappy
🤢	x_2	Unhealthy	Healthy
🥳	x_3	Noisy	Quiet

Grover's algorithm



$$f(x_0, x_1, x_2, x_3) =$$

$$(x_2 \vee x_0 \vee x_3) \wedge (\bar{x}_0 \vee x_3 \vee x_1) \wedge$$

$$(x_2 \vee \bar{x}_3 \vee x_1) \wedge (\bar{x}_1 \vee x_3 \vee \bar{x}_2) \wedge$$

$$(\bar{x}_0 \vee x_2 \vee \bar{x}_1) \wedge (x_1 \vee \bar{x}_2 \vee x_0) \wedge$$

$$(\bar{x}_0 \vee \bar{x}_3 \vee \bar{x}_2$$

Can find states which satisfy the logical statement f in $O(\sqrt{N})$ queries

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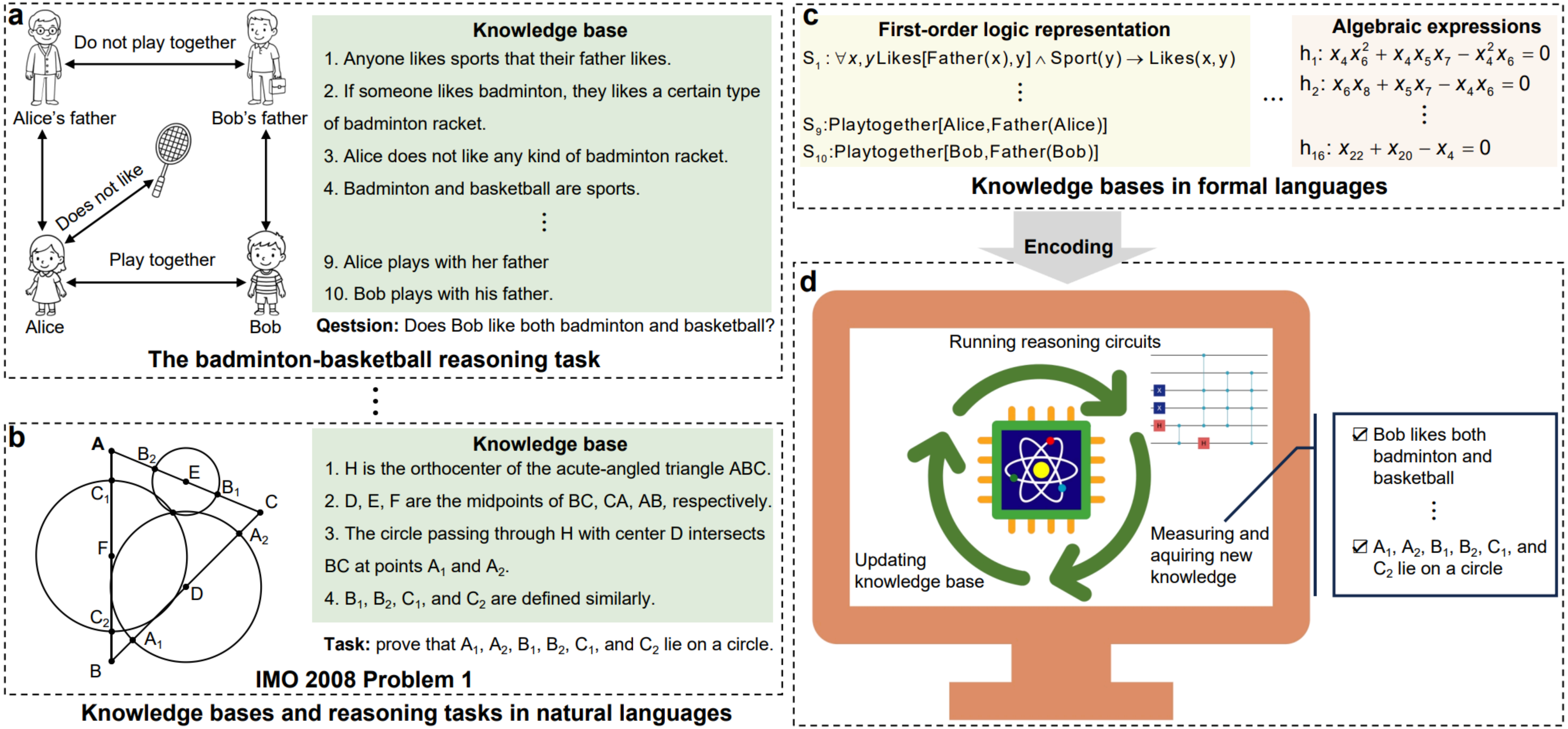
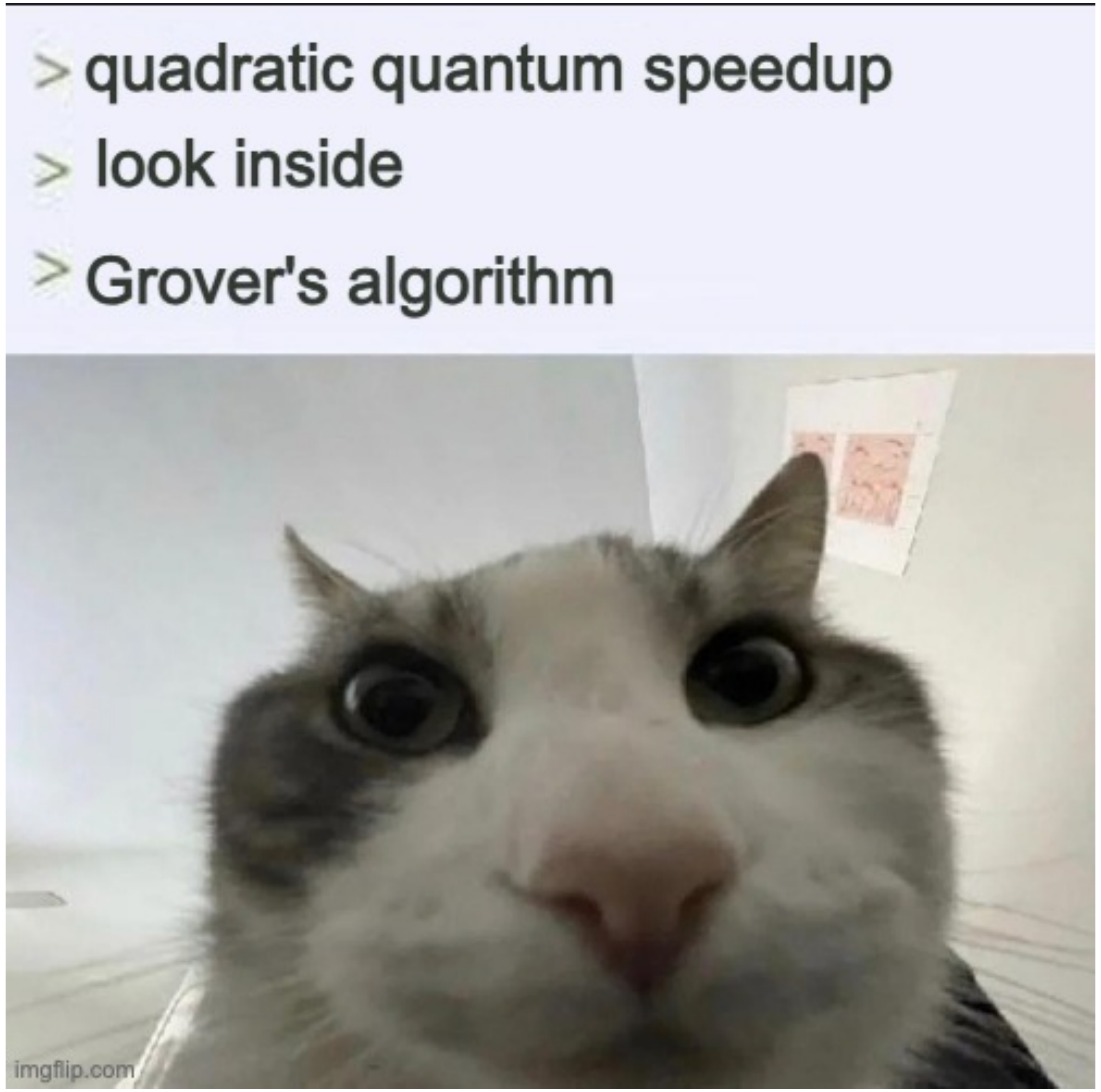


Conclusion

Conclusion



> quadratic quantum speedup
> look inside
> Grover's algorithm
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