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QARS Olivier · Majorana and Fermionic Gaussian states

1. Majoran Operators
2. Majoran Strings and group structure
3. Parity Postselection
4. Majorana Cliffords and Braids
5. Fermionic Gaussian States

N fermionic modes 2N operators c_i, $c_i^\dagger$

$$\{c_i, c_j^\dagger\} = \delta_{ij}$$

$$\{c_i^\dagger, c_j^\dagger\} = 0$$

$$\{c_i, c_j\} = 0$$

define vacuum $|0\rangle$ s.t $c_i|0\rangle = 0 \quad \forall i$

$c_i^\dagger|i\rangle = i$

$c_i^\dagger c_j^\dagger |0\rangle = |ij\rangle - |ji\rangle$

-> Pauli principle

define Majorana operators: $\boxed{\begin{aligned} \gamma_{2j} &= c_j + c_j^\dagger \\ \gamma_{2j+1} &= i(c_j - c_j^\dagger) \end{aligned}}$

new properties

$$\boxed{\begin{aligned} &\gamma_j^\dagger = \gamma_j \quad \forall j \\ &\gamma_j^2 = \mathbb{1} \\ &\{\gamma_i, \gamma_j\} = 2\delta_{ij} \end{aligned}}$$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$(c_j + c_j^\dagger)(c_j + c_j^\dagger) = \underbrace{c_j c_j}_{=0} + \underbrace{c_j^\dagger c_j + c_j c_j^\dagger}_{=\{c_j^\dagger, c_j\}} + \underbrace{c_j^{\dagger 2}}_{0}$$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

Majorana String

$$\mu(\vec{v}) = \underbrace{(i)^{v^T \omega_L v}}_{\text{convenient algebraic}} \gamma_0^{v_0} \gamma_1^{v_1} \dots \gamma_{2N}^{v_{2N}}$$

Finite field (bitstring)

$v \in \mathbb{F}_2^{2N}$

$(1 \dots 0) \rightarrow \gamma_0$

$(11 \dots 0) \rightarrow \gamma_0 \gamma_1$

$$\omega_L = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & 1 & 0 \end{pmatrix}$$

symplectic product $\langle v, v' \rangle_L = v^T \omega_L v'$

not local: Jordan-Wigner leads to non local Pauli strings. Pauli strings use a different symplectic matrix

Majorana group: majorana strings form a group

$$M_{2N} = \{ (i)^a \mu(v) \mid a \in \mathbb{Z}_4, \vec{v} \in \mathbb{F}_2^{2N} \}$$

$$\mu(\vec{v})\mu(\vec{v}') = \underset{\text{phase}}{\xi(v,v')}\, \mu(\underset{\text{XOR}}{v+v'})$$

if no majoranas in common, just reorder. Otherwise $\gamma_i^2 = \mathbb{1}$ disapear:

$$\mu(\vec{v})\mu(\vec{v}') = (-1)^{v^T \omega v'} \mu(\vec{v}')\mu(\vec{v})$$

$$\omega = \overleftrightarrow{1} - \mathbb{1} = \begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & & \vdots \\ 1 & 1 & \ddots & & 1 \\ \vdots & & & 0 & \\ 1 & 1 & \cdots & & 0 \end{pmatrix}$$

$$\langle v, v' \rangle \equiv v^T \omega v' = v^T \omega_L v' + v^T \omega_L^T v'$$

Max: same as Pauli Strings

define some brackets (commutators)

$$[\mu, \mu'] = \mu\mu' - \mu'\mu$$

defines a Lie algebra. We are looking for a sub algebra

$$[\gamma_i, \gamma_j] = 0 \quad \text{if} \quad i = j$$

$$\gamma_i \gamma_j - \gamma_j \gamma_i = 2\gamma_i \gamma_j \quad i \neq j$$

$$[\gamma_i, \gamma_j\gamma_k] = \begin{cases} 0 & \text{if } i=j=l \text{ or } i=k=l \\ & i \neq j \neq k \end{cases}$$

$$\gamma_i \gamma_j \gamma_k - \gamma_j \gamma_k \gamma_i = 0$$

$$\gamma_i \gamma_i \gamma_k - \gamma_i \gamma_k \gamma_i = 2\gamma_k$$

$\vdots$

au propre:

$$\cancel{[\gamma_i, \gamma_j\gamma_k]} \qquad [\gamma_i, \gamma_j\gamma_k] = \begin{cases} 2\gamma_k & i=j\neq k \\ 0 & i\neq j\neq k \end{cases}$$

$$[\gamma_i\gamma_j, \gamma_k\gamma_l] = \begin{cases} 0 & i\neq j\neq k\neq l : 0 \\ \gamma_i\gamma_k & i\neq k\neq j=l \end{cases}$$

quadratic majorana are closed (sub algebra)
free fermionic Hamiltonian $\binom{2N}{N}$ → quadratic number of operators in that space

$$H = \sum_{ij} \alpha_{ij} \gamma_i \gamma_j$$

What if you add one quartic terms

$$[\gamma_i \gamma_j \gamma_k \gamma_l, \gamma_m \gamma_n \gamma_o \gamma_p] \rightarrow \begin{matrix} 2 \\ 4 \\ \underline{\underline{6}} \end{matrix}$$

Start generating full algebra No

Parity Superselection: define weight

$w(\mu(\vec{v}))$: Hamming weight of $\vec{v}$

$P(\mu(\vec{v}))$: parity of $w(\mu(\vec{v}))$

Parity operator:

$$(-1)^{\hat{w}} = \mu(\vec{1})$$

$\hat{w}$ ← weight operator

$[O, \mu(\vec{1})] = 0$

↳ $|\text{even}\rangle + |\text{odd}\rangle$ $\emptyset$

Superselection rule ⟵ can't exist

if $\langle\psi_0|H|\psi_1\rangle = 0$ for some H $\equiv$ selection rule

if $\langle\psi_0|H|\psi_1\rangle = 0$ for all operators $\equiv$ superselection

No physical operators that

$$|\psi\rangle = (1 + c_B^\dagger)|0\rangle$$

Alice sends 0 : apply $\mathbb{1}$

1 : apply $i(c_A^\dagger - c_A)$

This would allow faster than light communication because of the ~~Fer~~ Pauli exclusion

Bob measures $\hat{O} = (1 + c_B + c_B^\dagger)$

gets 1 if Alice prepared 0

0 if Alice prepared $\emptyset$