A. No Single-Copy Purification Channel : Exists no quantum channel $\mathcal{M}_{\text{1-purif}}$ such that

$$\mathcal{M}_{\text{1-purif}}(\rho) = |\rho\rangle\langle\rho| . \tag{1}$$

Linearity :

$$\mathcal{M}_{\text{1-purif}}\left(\frac{1}{2}\rho_1 + \frac{1}{2}\rho_2\right) = \frac{1}{2}|\rho_1\rangle\langle\rho_1| + \frac{1}{2}|\rho_2\rangle\langle\rho_2| . \tag{2}$$

B. Purification Channel : Exists quantum channel $\mathcal{N}_{\text{purif}}$ such that

$$\mathcal{N}_{\text{purif}}\left(\rho^{\otimes n}\right) = \mathbb{E}_{|\rho\rangle}|\rho\rangle\langle\rho|^{\otimes n} . \tag{3}$$

C. Pure State Tomography Reduction : Take state of the art pure state tomography algorithm $\mathcal{A}$:

$$\mathcal{A}\left(\mathbb{E}_{|\rho\rangle}|\rho\rangle\langle\rho|^{\otimes n}\right) \approx \mathbb{E}_{|\rho\rangle}|\rho\rangle\langle\rho| \tag{4}$$

$$\text{Tr}_{\text{purif}}\left(\mathbb{E}_{|\rho\rangle}|\rho\rangle\langle\rho|\right) = \mathbb{E}_{|\rho\rangle}\rho = \rho \tag{5}$$

D. Summary of Results :

- First optimal full mixed state tomography : $O\left(\frac{1}{\epsilon}\left[rd + \log(\frac{1}{\delta})\right]\right)$. (precision ϵ, rank r, dim d, success prob. $1-\delta$).
- Other tomography applications (limited-entanglement, shadow tomography, quantum metrology).