Minimum Weight Decoding in the Colour Code is NP-hard

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1. Preliminaries:
- Quantum error correction codes on a lattice.
- A decoder finds the smallest set of errors that generates a syndrome.

2. Paper result :
Decoding hexagonal colour code is NP-hard.

3. Proof idea :
Given an instance F of 3-SAT, they create a syndrome S with $|S|$ defects such that:

- Each error set is at least size $|S|$.
- An error set of size $|S|$ exists $\Leftrightarrow$ F is true.

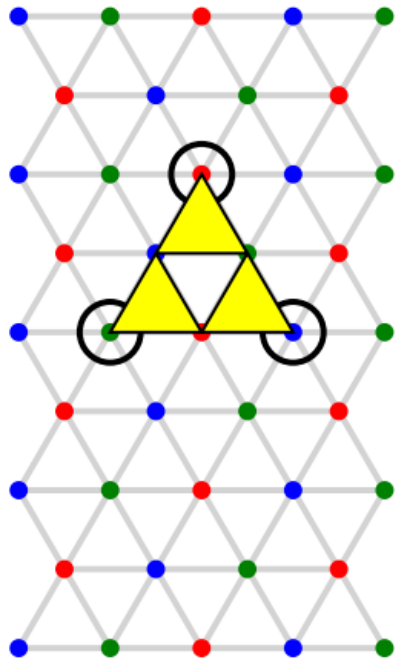

Figure: Circled nodes are defects of errors on yellow qubits.