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Product universal

$$\langle \mathcal{B} \rangle_{\times} = \bigcup_{r=1}^{\infty} \{ B_{i_1} \cdots B_{i_r} \mid (B_{i_1}, \dots, B_{i_r}) \in \mathcal{B}^r \}$$

$$\text{basis}[\langle \mathcal{B} \rangle_{\times}] = \mathcal{P}_N \quad \min |\mathcal{B}| = 2N$$

Adjoint universal

$$\langle \mathcal{A} \rangle_{\text{Lie}} = \text{span}_{\mathbb{R}} \{ \bigcup_{r=0}^{\infty} \mathcal{A}_{\text{ad}^{(r)}} \}$$

$$\mathcal{A}_{\text{ad}^{(r)}} = \{ \text{ad}_{\mathbf{A}}^r(A) \mid (\mathbf{A}, A) \in \mathcal{A}^{r+1} \}$$

$$\text{ad}_{\mathbf{A}}^r(A) = \underbrace{[A_{i_r}, \cdots, [A_{i_1}, A] \cdots]}_{\times r}$$

$$\langle \mathcal{A} \rangle_{\text{Lie}} = \mathfrak{su}(2^N) \quad \min |\mathcal{A}| = 2N + 1$$

